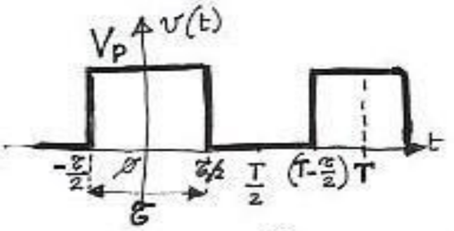


$\tau/T = \frac{1}{2}$ $\quad f_0 = \frac{1}{T} = \frac{\omega_0}{2\pi}$

$\frac{\tau}{2} = \frac{T}{4}$ $\quad \omega_0 = \frac{2\pi}{T}$

ONDA QUADRA UNIPOLARE POSITIVA, PARI

$A_K = \varnothing \quad \forall K$, perchè $v(t)$ è PARI

$$v(t) = C_0 + \sum_{K=1}^{\infty} A_K \sin(K\omega_0 t) + \sum_{K=1}^{\infty} B_K \cos(K\omega_0 t)$$

$$\boxed{C_0 = V_P \frac{\tau}{T} = \frac{V_P}{2}}$$

oppure $C_0 = \frac{1}{T}\int_{-\tau/2}^{T-\tau/2} v(t)\,dt = \frac{1}{T}\left[\int_{-\tau/2}^{\tau/2} V_P\,dt + \int_{\tau/2}^{T-\tau/2} \varnothing\,dt\right] =$

$$= \frac{1}{T}\,2\int_0^{\tau/2} V_P\,dt = \frac{2V_P}{T}\left[t\right]_0^{\tau/2} = \frac{2V_P}{T}\frac{T}{4\cdot 2} = \boxed{\frac{V_P}{2}}$$

$$B_K = \frac{2}{T}\int_{-\tau/2}^{T-\tau/2} v(t)\cos(K\omega_0 t)\,dt = \frac{2}{T}\left[\int_{-\tau/2}^{\tau/2} V_P\cos(K\omega_0 t)\,dt + \int_{\tau/2}^{T-\tau/2}\varnothing\,dt\right] =$$

$$= \frac{2V_P\,2}{T}\int_0^{\tau/2}\cos(K\omega_0 t)\,dt = \frac{4V_P}{T}\left[\frac{\sin(K\omega_0 t)}{K\omega_0}\right]_0^{\tau/2} = \frac{4V_P}{K\frac{2\pi}{T}T}\sin\left(K\frac{2\pi}{T}\frac{T}{4}\right) =$$

$$= \boxed{\frac{2V_P}{K\pi/2}\sin(K\pi/2)}$$

$B_1 = \frac{2V_P}{\pi}\sin\left(\frac{\pi}{2}\right) = 2V_P/\pi$

$B_2 = \frac{2V_P}{2\pi}\sin(\pi) = \varnothing$

$B_3 = \frac{2V_P}{3\pi}\sin\left(\frac{3\pi}{2}\right) = -2V_P/3\pi$

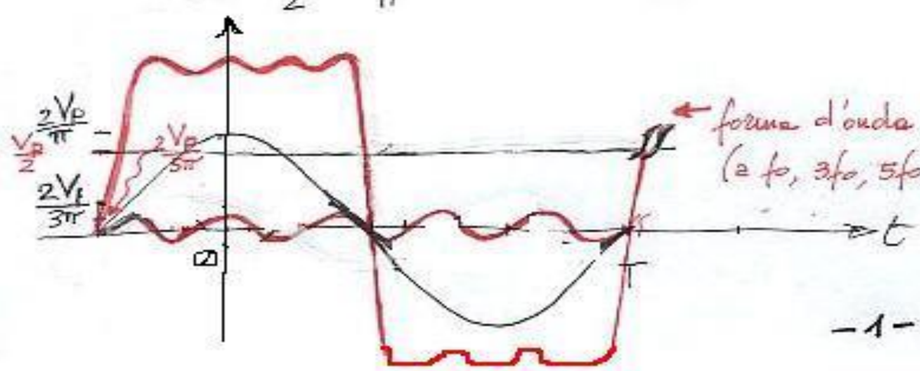
$B_4 = \varnothing$

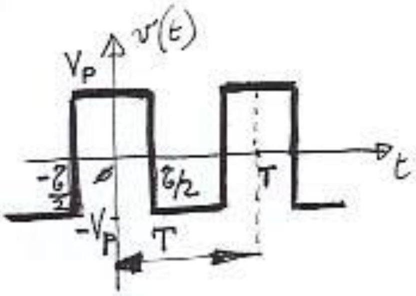
$B_5 = +2V_P/5\pi$

$B_6 = \varnothing$

$B_7 = -2V_P/7\pi$ $\qquad B_8 = \varnothing$

$$v(t) = \frac{V_P}{2} + \frac{2V_P}{\pi}\cos(2\pi f_0 t) + \frac{2V_P}{3\pi}\cos(2\pi\cdot 3f_0 t) + \frac{2V_P}{5\pi}\cos(2\pi\, 5f_0 t) + \frac{2\pi}{7\pi}\cos(2\pi 7f_0 t) + \dots$$





$\tau \equiv T_H = \frac{T}{2}$ $\left(\frac{\tau}{2} = \frac{T_H}{2} = \frac{T}{4}\right)$ $\left(\tau/T = \frac{1}{2}\right)$ $f_0 = \frac{1}{T}$

ONDA QUADRA ALTERNATA, PARI

$$C_0 = V_{PP}\frac{T_H}{T} - V_P = 2V_P\frac{1}{2} - V_P = \emptyset$$

$A_K = \emptyset \quad \forall K$

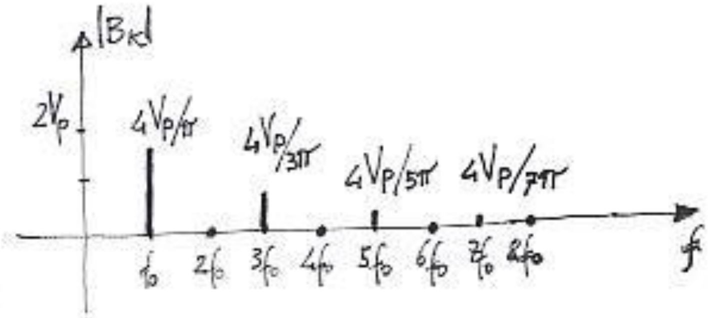
$$B_K = \frac{2}{T}\int_{-\tau/2}^{T-\tau/2} v(t)\cos(K\omega_0 t) = \frac{2}{T}\left[\int_{-\tau/2}^{\tau/2} V_P\cos(K\omega_0 t)dt + \int_{\tau/2}^{T-\tau/2} -V_P\cos(K\omega_0 t)dt\right] =$$

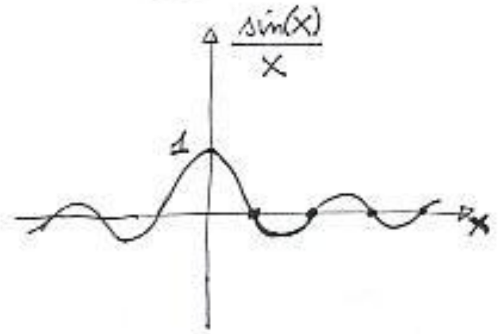
$$= \frac{2V_P}{T}\left[2\int_{\emptyset}^{T/4}\cos(K\omega_0 t)dt - 2\int_{T/4}^{T/2}\cos(K\omega_0 t)dt\right] = \frac{4V_P}{T}\left[\left[\frac{\sin(K\omega_0 t)}{K\omega_0}\right]_{\emptyset}^{T/4} - \left[\frac{\sin(K\omega_0 t)}{K\omega_0}\right]_{T/4}^{T/2}\right] =$$

$$= \frac{4V_P}{K\omega_0 T}\left[\sin\left(K\omega_0\frac{T}{4}\right) - \emptyset - \sin\left(K\omega_0\frac{T}{2}\right) + \sin\left(K\omega_0\frac{T}{4}\right)\right] =$$

$$= \frac{\cancel{4}^{2} V_P}{K\frac{2\pi}{\cancel{T}}\cancel{T}}\left[\sin\left(K\frac{2\pi}{T}\frac{T}{4}\right) - \sin\left(K\frac{2\pi}{T}\frac{T}{2}\right) + \sin\left(K\frac{2\pi}{T}\frac{T}{4}\right)\right] = \frac{2V_P}{K\pi}\left[\sin\left(\frac{K\pi}{2}\right) - \underbrace{\sin(K\pi)}_{=\emptyset} + \sin\left(\frac{K\pi}{2}\right)\right] =$$

$$= \frac{4V_P}{K\pi}\left[\sin\left(\frac{K\pi}{2}\right)\right] = \frac{4V_P\cdot\frac{1}{2}}{K\pi\frac{1}{2}}\sin\left(K\pi\frac{1}{2}\right) = \boxed{2V_P\frac{\sin\left(\frac{K\pi}{2}\right)}{\frac{K\pi}{2}}} \qquad \boxed{4V_P\frac{\tau}{T}\frac{\sin(K\pi\tau/T)}{K\pi\tau/T}}$$





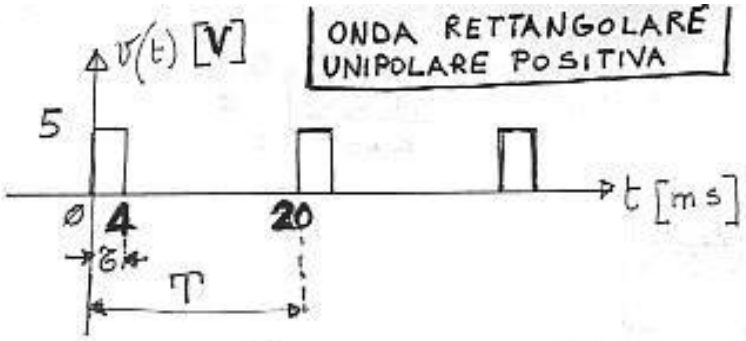
$f_{z_1} = \frac{1}{\tau} = \frac{1}{\frac{T}{2}} = \frac{2}{T} = 2f_0$

$f_{z_2} = \frac{2}{\tau} = \frac{2}{\frac{T}{2}} = \frac{4}{T} = 4f_0$

$$\begin{cases} B_1 = 2V_P\dfrac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \dfrac{4V_P}{\pi} \\ B_2 = \emptyset \\ B_3 = 2V_P\dfrac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} = -\dfrac{4V_P}{3\pi} \\ B_4 = \emptyset \\ B_5 = 4V_P/5\pi \\ B_6 = 0 \\ B_7 = -4V_P/7\pi \\ B_8 = \emptyset \end{cases}$$

$$v(t) = \frac{4V_P}{\pi}\cos(2\pi f_0 t) - \frac{4V_P}{3\pi}\cos(2\pi 3f_0 t) + \frac{4V_P}{5\pi}\cos(2\pi 5f_0 t) + \ldots \ldots \quad [V]$$

ONDA RETTANGOLARE UNIPOLARE POSITIVA



$$\tau = 4\,[ms] \qquad T = 20\,[ms] \qquad \boxed{\tau / T = \frac{1}{5}}$$

$$\omega_0 = \frac{2\pi}{T} \qquad f_0 = \frac{1}{T} = 50\,[Hz]$$

$$v(t) = C_0 + \sum_{K=1}^{\infty} A_K \sin(K\omega_0 t) + \sum_{K=1}^{\infty} B_K \cos(K\omega_0 t)$$

$$C_0 = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\left[\int_0^{\tau} 5\,dt + \int_{\tau}^{T} \varnothing\,dt\right] = \frac{1}{T}\cdot 5\int_0^{\tau} dt = \frac{5}{T}\left[t\right]_0^{\tau} = \frac{5}{T}\left[\tau\right] = 5\cdot\frac{4}{20} = 1\,[V]$$

$$\left(C_0 = V_{MAX}\cdot\frac{\tau}{T} = 5\cdot\frac{1}{5} = 1\right)$$

$$A_K = \frac{2}{T}\int_0^T v(t)\sin(K\omega_0 t)\,dt = \frac{2}{T}\int_0^{\tau} 5\sin(K\omega_0 t)\,dt = \frac{10}{T}\left[-\frac{\cos(K\omega_0 t)}{K\omega_0}\right]_0^{\tau} =$$

$$= \frac{10}{K\omega_0 T}\left[-\cos(K\omega_0\tau) + \cos(\varnothing)\right] = \frac{\cancel{10}\,5}{K\frac{2\pi}{\cancel{T}}\cancel{T}}\left[1 - \cos\left(K\frac{2\pi}{T}\tau\right)\right] =$$

$$\boxed{= \frac{5}{K\pi}\left[1 - \cos\left(K\frac{2\pi}{5}\right)\right]}$$

$$A_1 = \frac{5}{\pi}\left[1 - \cos\left(\frac{2\pi}{5}\right)\right] \simeq 1{,}1$$

$$A_2 = \frac{5}{2\pi}\left[1 - \cos\left(\frac{4\pi}{5}\right)\right] \simeq 1{,}44$$

$$A_3 = \frac{5}{3\pi}\left[1 - \cos\left(\frac{6\pi}{5}\right)\right] \simeq 0{,}96$$

$$A_4 = \frac{5}{4\pi}\left[1 - \cos\left(\frac{8\pi}{5}\right)\right] \simeq 0{,}27$$

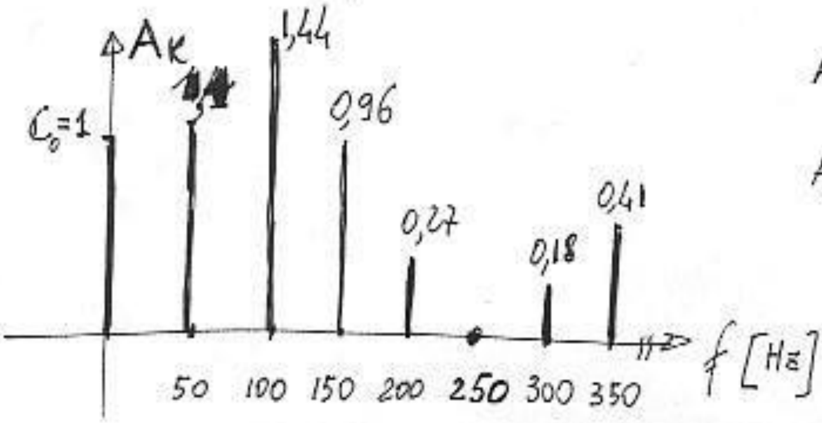
$$A_5 = \frac{5}{5\pi}\left[1 - \cos\left(\frac{10\pi}{5}\right)\right] \simeq \varnothing$$

$$A_6 = \frac{5}{6\pi}\left[1 - \cos\left(\frac{12\pi}{5}\right)\right] \simeq 0{,}18$$

$$A_7 = \frac{5}{7\pi}\left[1 - \cos\left(\frac{7\pi}{5}\right)\right] = 0{,}41$$

serie dei seni:

$$1{,}1\,\sin(2\pi\cdot 50t) + 1{,}44\,\sin(2\pi\cdot 2\cdot 50t) +$$
$$+ 0{,}96\,\sin(2\pi\cdot 3\cdot 50t) + 0{,}27\,\sin(2\pi\cdot 4\cdot 50t) + \ldots\ldots$$



$$B_K = \frac{2}{T}\int_0^{\tau} 5\cos(K\omega_0 t)\,dt = \frac{10}{T}\int_0^{\tau}\cos(K\omega_0 t)\,dt = \frac{10}{T}\left[\frac{\sin(K\omega_0 t)}{K\omega_0}\right]_0^{\tau} =$$

$$= \frac{10\,\cancel{5}}{K\frac{2\pi}{\cancel{T}}\cancel{T}}\left[\sin\left(\frac{K2\pi}{T}\tau\right)\right] = \boxed{\frac{5}{K\pi}\sin\left(\frac{K2\pi}{5}\right)}$$

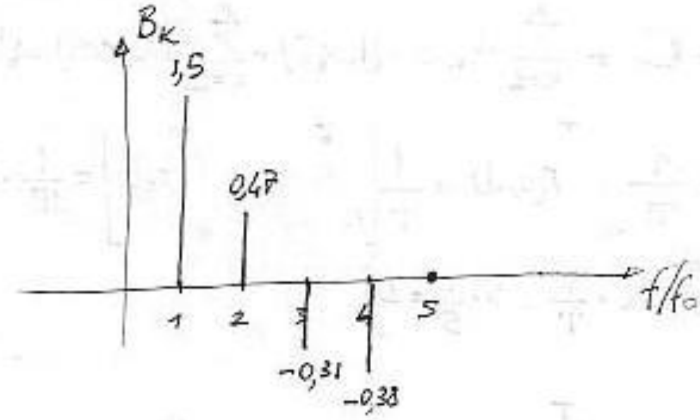
$B_1 = \frac{5}{\pi}\sin\left(\frac{2\pi}{5}\right) \cong 1{,}5$

$B_2 = \frac{5}{2\pi}\sin\left(\frac{4\pi}{5}\right) \cong 0{,}47$

$B_3 = \frac{5}{3\pi}\sin\left(\frac{6\pi}{5}\right) = -0{,}31$

$B_4 = \frac{5}{4\pi}\sin\left(\frac{8\pi}{5}\right) = -0{,}38$

$B_5 = \frac{5}{5\pi}\sin\left(\frac{10\pi}{5}\right) = \varnothing$

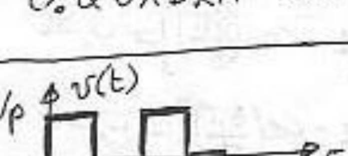


serie dei coseni: $1{,}5\cos(2\pi 50t) + 0{,}47\cos(2\pi 100t) - 0{,}31\cos(2\pi 150t) - 0{,}38\cos(2\pi 200t) + \dots$

$v(t) = 1 + 1{,}1\sin(2\pi 50t) + 1{,}44\sin(2\pi 100t) + 0{,}96\sin(2\pi 150t) + 0{,}275\sin(2\pi 200t) + \dots + 1{,}5\cos(2\pi 50t) + 0{,}47\cos(2\pi 100t) - 0{,}31\cos(2\pi 150t) - 0{,}38\cos(2\pi 200t) + \dots$

— o — o —

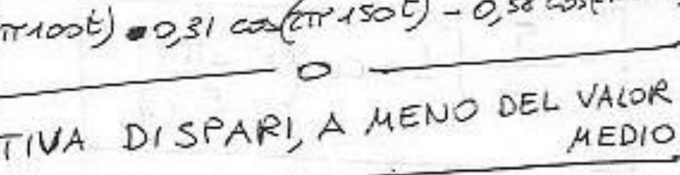
O. QUADRA UNIPOLARE POSITIVA DISPARI, A MENO DEL VALOR MEDIO



$$C_0 = V_p\,\tau/T = V_p/2$$

$$A_K = \frac{2}{T}\left\{\int_0^{T/2} V_p\sin(K\omega_0 t)\,dt + \int_{T/2}^{T}\varnothing\,dt\right\} = \frac{2V_p}{T}\int_0^{T/2}\sin(K\omega_0 t)\,dt = \frac{2V_p}{T}\left[-\frac{\cos(K\omega_0 t)}{K\omega_0}\right]_0^{T/2} =$$

$$= \frac{2V_p}{K\frac{2\pi}{\cancel{T}}\cancel{T}}\left[-\cos\left(K\frac{2\pi}{\cancel{T}}\frac{\cancel{T}}{2}\right) + \cos\left(K\frac{2\pi}{T}\varnothing\right)\right] = \frac{V_p}{K\pi}\left[1 - \cos(K\pi)\right]$$



$A_1 = \frac{2V_p}{\pi}$ $\quad A_6 = \varnothing$

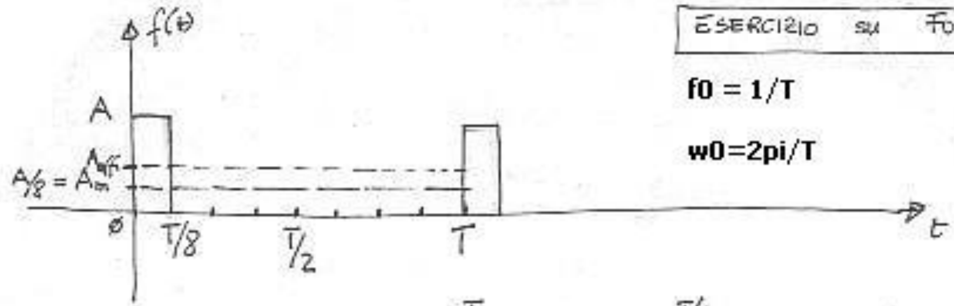
$A_2 = \varnothing$

$A_3 = \frac{2V_p}{3\pi}$

$A_4 = \varnothing$

$A_5 = \frac{2V_p}{5\pi}$

$$\boxed{B_K = \varnothing\ \forall K}$$



ESERCIZIO SU FOURIER

f0 = 1/T

w0=2pi/T

C_0

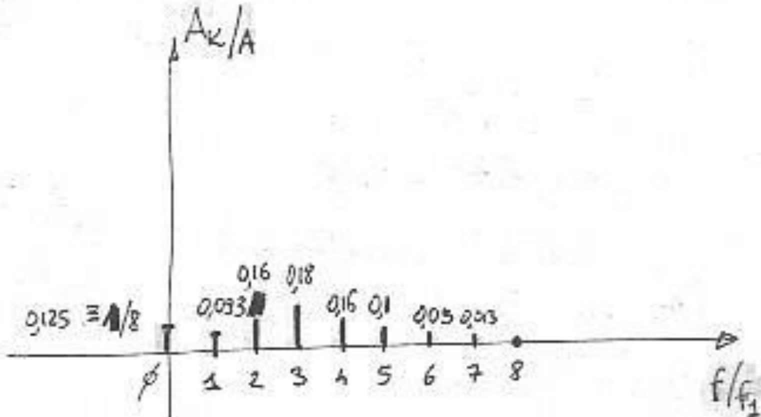
a) valor medio $f_m(t) = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\int_0^{T/8} A\,dt = \frac{A}{T}[t]_0^{T/8} = \frac{A}{T}\cdot\frac{T}{8} = \boxed{\frac{A}{8}} \simeq 12{,}5\%\,A$

b) v. eff. $\triangleq \sqrt{v.q.m} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} = \sqrt{\frac{1}{T}\int_0^{T/8} A^2 dt} = \sqrt{\frac{A^2}{T}[t]_0^{T/8}} = \sqrt{\frac{A^2}{T}\cdot\frac{T}{8}} = \sqrt{\frac{A^2}{8}} = \boxed{\frac{A}{2}\frac{1}{\sqrt{2}}} = 35\%\,A$

c) $A_K = \frac{2}{T}\int_0^T \mathbf{f(t)\sin(kw0t)dt} = \frac{2}{T}\int_0^{T/8} \mathbf{A\sin(kwot)dt} = \frac{2A}{T}\left[\frac{-\cos(\mathbf{kw0t})}{K\omega}\right]_{\mathbf{0}}^{\mathbf{T/8}} = \frac{2A}{K\omega T}\left[-\cos K\omega\frac{T}{8} + \cos\phi\right] =$

$= \frac{2A}{K\frac{2\pi}{T}T}\left[-\cos K\frac{2\pi}{T}\frac{T}{8_4} + 1\right] = \boxed{\frac{A}{K\pi}\left[1 - \cos K\frac{\pi}{4}\right]}$

$$\begin{cases}
A_1 = \frac{A}{\pi}\left[1 - \cos\frac{\pi}{4}\right] = 0{,}093\,A \\
A_2 = \frac{A}{2\pi}\left[1 - \underbrace{\cos\frac{\pi}{2}}_{=0}\right] = \frac{A}{2\pi} = 0{,}159\,A \\
A_3 = \frac{A}{3\pi}\left[1 - \cos\frac{3\pi}{4}\right] = 0{,}18\,A \\
A_4 = \frac{A}{4\pi}\left[1 - \cos\pi\right] = \frac{A}{2\pi} = 0{,}159\,A \\
A_5 = \frac{A}{5\pi}\left[1 - \cos\frac{5}{4}\pi\right] = 0{,}108\,A \\
A6 = \frac{A}{6\pi}\left[1 - \cos\frac{6}{4}\pi\right] = \frac{A}{6\pi} = 0{,}053\,A \\
A_7 = \frac{A}{7\pi}\left[1 - \cos\frac{7}{4}\pi\right] = 0{,}013\,A \\
A_8 = \frac{A}{8\pi}\left[1 - \cos 2\pi\right] = \phi
\end{cases}$$



Si annullano le righe il cui indice K è un multiplo dell'inverso del duty-cycle

cioè $K = 8, 16, 24, 32 \ldots\ldots$ $\left[K = \frac{n}{\frac{\tau}{T}} = \frac{n}{\frac{1}{8}}\right]$ con $n = 1, 2, \ldots$

d) $B_K = \mathbf{\frac{2}{T}}\int_0^T f(t)\cos(K\omega t)\,dt = \frac{2}{T}\int_0^{T/8} A\cos(K\omega t)\,dt = \frac{2A}{T}\left[\frac{\sin(K\omega t)}{K\omega}\right]_0^{T/8} = \frac{2A}{K\frac{2\pi}{T}T}\left[\sin\left(K\frac{2\pi}{T}\frac{T}{8_4}\right) - \phi\right] = \boxed{\frac{A}{K\pi}\sin\left(K\frac{\pi}{4}\right)}$

$B_1 = \frac{A}{\pi} \sin\frac{\pi}{4} = 0{,}23A$

$B_2 = \frac{A}{2\pi} \sin\frac{\pi}{2} = 0{,}16A$

$B_3 = \frac{A}{3\pi} \sin\frac{3\pi}{4} = 0{,}08A$

$B_4 = \frac{A}{4\pi} \sin\pi = 0$

$B_5 = \frac{A}{5\pi} \sin\frac{5}{4}\pi = -0{,}05A$

$B_6 = \frac{A}{6\pi} \sin\frac{6\pi}{4} = -0{,}05A$

$B_7 = \frac{A}{7\pi} \sin\frac{7}{4}\pi = -0{,}03A$

$B_8 = \frac{A}{8\pi} \sin\frac{8}{4}\pi = 0$

e) f(t) = 0,125A + 0,09A*sin(w0t) + 0,23A*cos(w0t) + 0,16A*sin(2w0t) + 0,16cos(2w0t) + ... [V]

f) $f(t) = \boxed{C_0 + \sum_K C_K \sin(K\omega t + \varphi_K)}$

$$\begin{cases} C_K = \sqrt{A_K^2 + B_K^2} \\ \varphi_K = \arctan\frac{B_K}{A_K} \end{cases}$$

$$\begin{cases} C_1 = \sqrt{0{,}93^2 + 2{,}25^2} = 2{,}43 \\ \varphi_1 = \arctan\frac{2{,}25}{0{,}93} = 67{,}5° \end{cases}$$

$$\begin{cases} C_2 = \sqrt{1{,}59^2 + 1{,}59^2} = 1{,}59\sqrt{2} = 2{,}25 \\ \varphi_2 = \arctan\frac{1{,}59}{1{,}59} = 45° \end{cases}$$

$$\begin{cases} C_3 = \sqrt{1{,}8^2 + 0{,}75^2} = 1{,}95 \\ \varphi_3 = \arctan\frac{0{,}75}{1{,}8} = 22{,}6° \end{cases}$$

$$\begin{cases} C_4 = \sqrt{\emptyset + 1{,}59^2} = 1{,}59 \\ \varphi_4 = \arctan\frac{\emptyset}{1{,}59} = \emptyset° \end{cases}$$

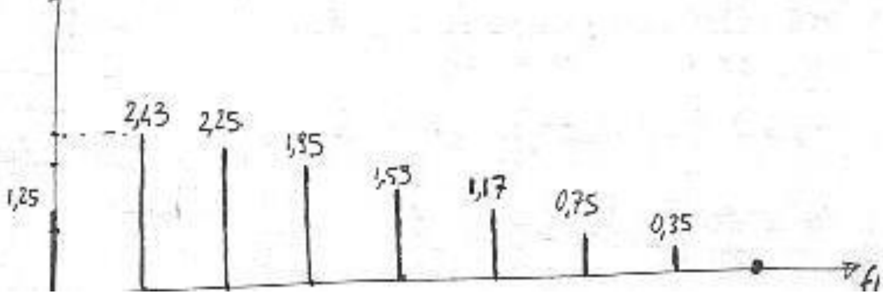
$$\begin{cases} C_5 = \sqrt{1{,}08^2 + 0{,}45^2} = 1{,}17 \\ \varphi_5 = \arctan\left(-\frac{0{,}45}{1{,}08}\right) = -22{,}6° \end{cases}$$

ponendo A = 10[V]

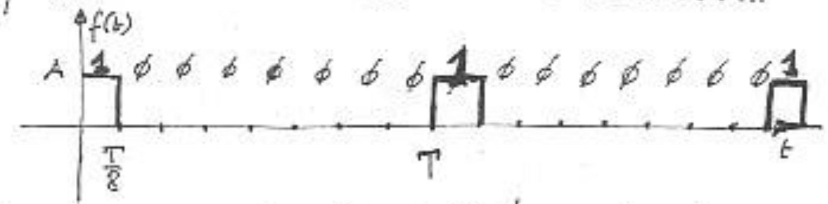
$f(t) = 1{,}25 + 2{,}43 \sin(\omega_1 t + 67{,}5°) + 2{,}25 \sin(2\omega_1 t + {}$
$+ 1{,}95 \sin(3\omega_1 t + 22{,}6°) + 1{,}59 \sin 4\omega_1 t$ ...[V]

Ck si annulla per k=8,16,24...

$$\begin{cases} C_8 = \sqrt{\emptyset + \emptyset} = \emptyset \\ \varphi_8 = \emptyset° \end{cases}$$



8) La $f(t)$ data può essere considerata come la sequenza binaria del bytes 1000000010000001......



si è visto come a questa forma d'onda corrisponde uno spettro dei C_K che si annulla per $K=8, 16, 24 \ldots$. Quindi il 1° lobo occupa la banda di frequenza $\phi \div 8 f_1$ cioè $\boxed{\phi \div 16\,[KHz]}$

Se la stessa sequenza binaria fosse trasmessa a velocità 10 volte maggiore, quanto si estenderebbe in frequenza il 1° lobo?

Per aumentare la velocità di un fattore 10, devo ridurre la durata del bit T_B di un fattore 10. (velocità di trasmissione ≡ N° di bit/sec)

Nel nostro esempio, in T secondi $= 0,5 \cdot 10^{-3}[s]$, $\longrightarrow \begin{cases} f = 2[KHz] \\ T = 0,5[ms] \end{cases}$

transitano 8 bit, la cui durata è $\frac{T}{8} = \frac{0,5}{8} \cdot 10^{-3} = 62,5[\mu s]$

Se, nello stesso tempo, ne devono passare 10 volte tanti, bisogna che $T_B = \frac{T}{80} = \frac{0,5}{80} = 6,25[\mu s]$. Però si deve ridurre della stessa quantità anche T, perché la forma d'onda non deve cambiare $[\delta = cost]$. T sarà pari a **50μs**, in modo che $T_{BIT} = 6,25\,\mu s = \frac{T}{8} = \frac{50}{8}\,\mu s$. Ciò significa, in termini di spettro, che la fondamentale sarà pari a $f = 20[KHz]$ e le armoniche superiori si localizzeranno nei multipli di tale valore (40, 60, 80 etc).

L'inviluppo dello spettro non cambia, l'altezza delle righe rimane invariata, aumenta, di un fattore 10, la distanza fra le righe. Il primo lobo quindi si estenderà da ϕ a $8 \cdot 20 = 160\,[KHz]$! La banda + significativa (1° lobo) si è allargata di un fattore 10.

Ecco perché aumentare la velocità di trasmissione può compromettere la tx str.. : può succedere che il canale non riesca a far passare le armoniche superiori!