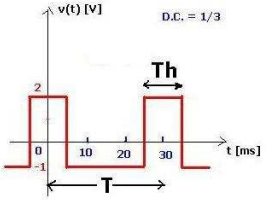
ESERCIZIO : FOURIER + FILTRO ATTIVO P. BASSO



D.C. = Th / T = 1/3

Th = T / 3 Th/2 = T/6

T = 30 [ms] f0 = 33 [Hz]

$$v(t) = C_0 + \sum_{K=1}^{\infty} B_K \cos(K\omega_0 t) \qquad A_K = \emptyset \quad \forall K$$

$$C_0 = 3 \cdot \frac{1}{3} - 1 = \emptyset$$

$$B_K = \frac{2}{T}\left[2\int_{\emptyset}^{T/6} 2\cos(K\omega_0 t)\,dt + \int_{T/6}^{5/6\,T} -1\cos(K\omega_0 t)\,dt \right] =$$

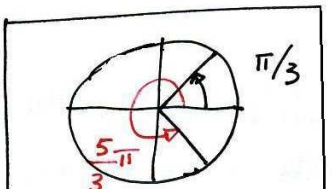
$$= \frac{2}{T}\left[4\,\frac{\sin(K\omega_0 t)}{K\omega_0}\Big|_{\emptyset}^{T/6} - \frac{\sin(K\omega_0 t)}{K\omega_0}\Big|_{T/6}^{\frac{5}{6}T} \right] =$$

$$= \frac{2}{K\omega_0 T}\left[4\sin(K\omega_0 T/6) - \sin\left(K\omega_0 \tfrac{5}{6}T\right) + \sin\left(K\omega_0 \tfrac{T}{6}\right) \right] =$$

$$= \frac{2}{K\frac{2\pi}{T}T}\left[4\sin\left(K\frac{2\pi}{T}\frac{T}{6}\right) - \sin\left(K\frac{2\pi}{T}\frac{5}{6}T\right) + \sin\left(K\frac{2\pi}{T}\frac{T}{6}\right) \right] =$$

$$= \frac{1}{K\pi}\left[4\sin\left(K\frac{\pi}{3}\right) - \sin\left(K\frac{5}{3}\pi\right) + \sin\left(K\frac{\pi}{3}\right) \right] =$$

$$= \frac{1}{K\pi}\left[6\sin(K\pi/3) \right] = \boxed{\frac{6}{K\pi}\sin(K\pi/3)}$$



$\pi/3 = 60°$

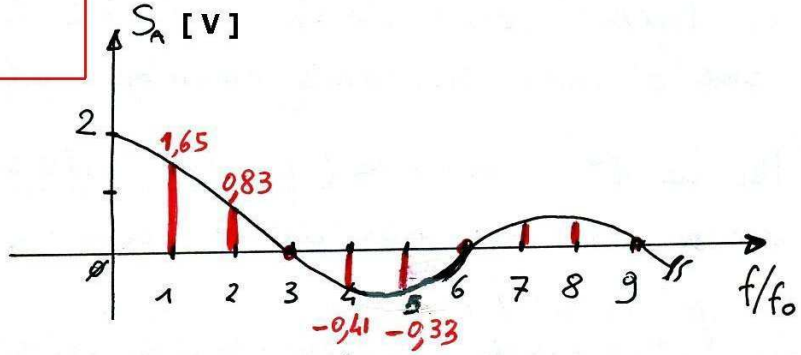
$\frac{5\pi}{3} = 300°$

$$\sin\left(\frac{\pi}{3}\right) = -\sin\left(\frac{5\pi}{3}\right)$$

FORMULA GENERALE (O. RETT. PARI)

$$B_K = 2 V_{pp}\,\tau/T\,\frac{\sin(K\pi\,\tau/T)}{K\pi\,\tau/T}$$

$$\begin{cases} B_1 = \frac{6}{\pi}\sin(K\pi/3) \simeq 1{,}65 \\ B_2 \simeq 0{,}83 \\ B_3 = \emptyset \\ B_4 = -0{,}41 \\ B_5 = -0{,}33 \\ B_6 = \emptyset \end{cases}$$

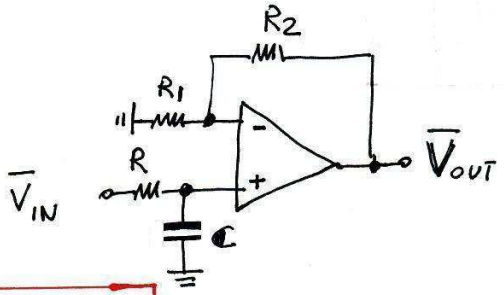


$$v(t) \simeq 1{,}65\cos(2\pi f_0 t) + 0{,}83\cos(2\pi\,2 f_0 t) - 0{,}41\cos(2\pi\,4 f_0 t) - 0{,}32\cos(2\pi\,5 f_0 t) + \dots$$

SPECIFICHE DEL FILTRO RC PASSA-BASSO ATTIVO NON INVERTENTE DEL 1° ORDINE

$$\begin{cases} G_{LF} = 6\ [dB] \\ f_t = 2 f_0 = 66\ [Hz] \end{cases}$$

Schema del Filtro :



$$\bar{G}(j\omega) = \frac{\bar{V}_{OUT}}{\bar{V}_{IN}} = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{1 + j\omega RC}$$

$GLF = 10^{6/20} \simeq 2$

$(Gmax = GLF)\ dB = 20\log(1+R2/R1) = 6\ [dB]$

$$1 + \frac{R_2}{R_1} = 2 \quad \Rightarrow \quad \frac{R_2}{R_1} = 1 \quad \Rightarrow \quad \begin{cases} R_1 = 10\ [K\Omega] \\ R_2 = 10\ \text{"} \end{cases} \quad (\text{per es.})$$

ft = 1 / 2piRC = 66 [Hz] pongo **C = 1[uF]**

R = 1/2pi*ft*C = 1/6,28*66 *10^-6 =2411[Ohm]

$$|G| = \frac{1 + R2/R1}{\sqrt{1 + (wRC)^2}}$$

Calcolo dei guadagni ricevuti dalle varie componenti armoniche del segnale, ad opera del filtro :
per un calcolo accurato bisogna sostituire, nell' espressione del |G|, a w i valori di w0, 2w0, 4w0, 5w0 ;
tenendo presente che wt = 1/RC = 2w0 cioè w0 = 1/2RC , si ottiene :

1) $|G(jw0)| = |G(j2*pi*33t)| = \frac{2}{\sqrt{1+0,25}} = 1,8$

2) $|G(j2w0) = |G(j2*pi*66t)| = \frac{2}{\sqrt{1+1}} = 1,4$

3) $|G(j4w0)| = |G(j2*pi*132t)| = \frac{2}{\sqrt{1+4}} = 0,9$

4) $|G(j5w0)| = |G(J2*pi*165t)| = \frac{2}{\sqrt{1+2,5^2}} = 0,7$

CALCOLO DELLA FASE

phi (w0) = - artan(1/2)= - 27°

phi(2w0) = -artan(1) = -45°

phi(4w0) = - artan(2) = - 63°

phi(5w0) = - artan(2,5) = - 68°

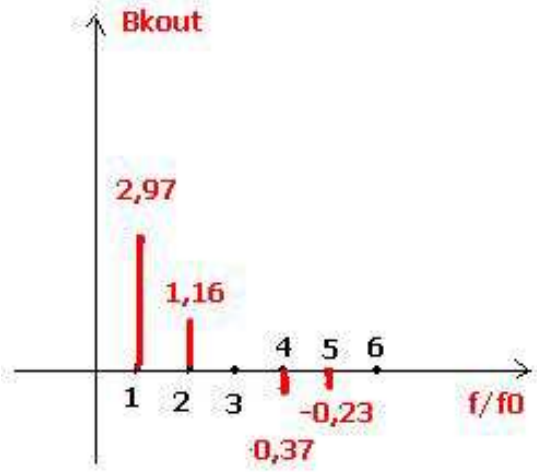
Perciò dopo il filtro le varie armoniche saranno alte :

B1out = B1in*|G(jw0)| = 1,65 * 1,8 = 2,97 [V]

B2out = 0,83 * 1,4 = 1,16 [V]

B4out = - 0,41 * 0,9 = - 0,37 "

B5out = - 0,33 * 0,7 = - 0,23 "



Spettro di ampiezza in uscita al filtro

La tensione in uscita al filtro è perciò :

$$v_{out}(t) = 2{,}97\cos(2\pi 33t - 27°) + 1{,}16\cos(2\pi 66t - 45°) - 0{,}37\cos(2\pi 132t - 63°) - 0{,}23\cos(2\pi 165t - 68°) + \ldots.$$